

IGCRE 2014

1st Joint Iran & Germany Conference on
Renewable Energy
14-RENE-123



September 9-10, 2014
Shahid Beheshti University, Tehran, Iran

Calculation of Wind Turbine Geometrical Angles Using Unsteady Blade Element Momentum (BEM)

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Abstract— Converting wind kinetic energy into electrical energy by using wind turbine is one of the interested approaches to get rid of fossil fuels as a source of energy. Wind turbine behavior's simulation is a common way to design a high performance turbine. One of the important parameters in wind turbine's design is the value of its geometrical angles. In this study we simulated the behavior of wind turbines by applying unsteady blade element method in order to calculate the appropriate values for these angles. Also we presented the effect of choosing appropriate value on the turbine's performance.

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Abstract—Converting wind kinetic energy into electrical energy by using wind turbine is one of the interested approaches to get rid of fossil fuels as a source of energy. Wind turbine behavior's simulation is a common way to design a high performance turbine. On of the important parameters in wind turbine's design is the value of it's geometrical angles. In this study we simulated the behavior of wind turbines by applying unsteady blade element method method in order to calculate the appropriate values for these angles. Also we presented the effect of choosing appropriate values on the turbine's performance.

I. INTRODUCTION

Wind power is the conversion of wind energy into a useful form of energy and wind turbine is a device that converts wind's kinetic energy into electrical energy. The main purpose of researches on wind turbines is to obtain generated power and providing methods to increase that.

Early research on wind turbines back to the nineteenth century. Where in 1865 Rankine for the first time expressed momentum theory for an actuator disk [1]. After, Froude [2] introduced blade element theory and the first time combined momentum theory and blade element theory called blade element momentum (BEM) theory. Then the maximum power can be obtained from a free stream which one specific application of is in wind turbines introduced. This theory is also named Lanchester (1915), Betz (1920) and Zhukovsky (1920) which all are based on Rankines momentum theory of actuator disk [3]. In 1926 Prandtl presented tip correction which is actually a correction for non-uniformity flow between blades [4]. A vortex wake description for induction presented by Goldstein in 1929 [5]. But the calculations were just for fixed, non-rotating vortices. Glauert introduced developed blade element momentum theory in 1935 that includes some aspects of wake swirl, but it's effect on radial pressure gradient were not considered [6]. In 1974 Wilson and Lissaman offered optimization of wind turbine revised by De Veris in 1979 and the aerodynamics of wind power machines and related fluid dynamics fields applied in [7]. In recent years many research projects, including engineering methods in order to dispel the limitations of the BEM theory has been presented.

Blade element momentum (BEM) method is one of the common methods to simulate the performance of a wind turbine which is simple and has less computation in comparison to

other methods. It is possible to calculate steady forces exerted on the blade by using this method. Also trust and power for different conditions of wind speed, rotational speed and blade angles can be calculated. To calculate the forces exerted at different times for a number of time-varying inputs other engineering models must also be included.

In the one-dimensional momentum theory real geometry of the rotor, number of blades, thickness changes, rotating blades and applied airfoil profile for blades were not considered. In order to evaluate the events surrounding the wind turbine's blade, definition of full geometry is necessary. Hansen et al. [8] proposed a geometry with four coordinate systems. These coordinate systems introduce various angles. As it will be shown, changing values of these angles has a direct impact on turbine performance. So the setting value of these angles is one of the factors that must be considered during design.

The purpose of this paper is to calculate the appropriate values for these angles and show the impact of choosing these new values on wind turbine performance.

II. THEORY

A. Geometry Definition

The first step to simulate the behavior of a wind turbine is to define it's geometry. Here we apply the method that Hansen et al. [8] applied to define the geometry of the wind turbine. This geometry is shown in figure 1.

Since wind characteristics at various times and places differ, it is important that the position of each point along the blade be specified relative to a fixed coordinate system at any moment. So fixed coordinate system (coordinate system 1) can be placed at the bottom of the tower. Coordinate system 2 is a non-rotating one and placed on the nacelle. This system is first rotated about the x -axis with the angle θ_{yaw} and then is rotated θ_{tilt} about the y -axis. Coordinate system 3 is mounted on the rotating shaft and rotated about the z -axis with the angle of θ_{wing} relative to the system 2. Coordinate system 4 is placed on one of the blades and only rotated θ_{cone} about the y -axis relative to the system 3. So vectors in each coordinate system can be easily transferred to another coordinate system by utilizing transformation matrix.

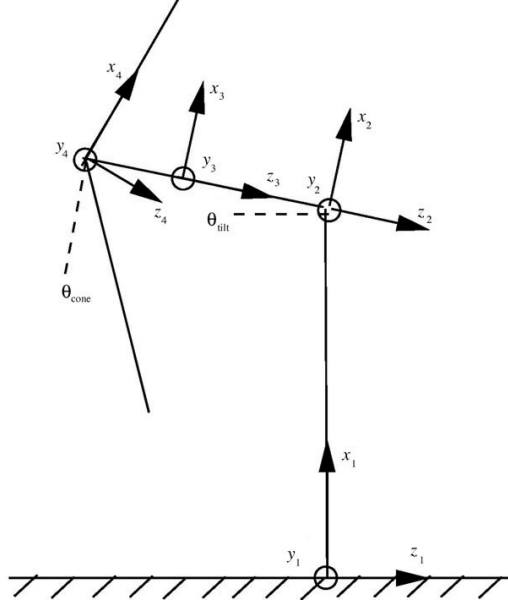


Fig. 1. Coordinate systems used for describing wind turbine geometry [8]

B. Unsteady BEM Method

Before describing BEM method it is reasonable to introduce simple one dimensional (1-D) method for an ideal rotor. In this model the rotor assumed as a permeable disk. Due to the negligible effect of friction and rotational velocity component in the wake, this disk considered as an ideal one. This device decreasing the wind speed from V_0 , far upstream, to the u , at the rotor plane, and u_1 , in the wake. Since the value of the Mach number is small, it is possible to assume incompressible flow, thus the density is constant along a streamline and the axial velocity reduces continuously. By assuming the rotor as an ideal disk, a simple relationship can be derived between velocities V_0 , u_1 and u [8]:

$$u = \frac{1}{2}(V_0 + u_1) \quad (1)$$

It is obvious that the velocity at the rotor plane is the mean of wind velocity V_0 and the wake velocity u_1 . By defining the axial induction factor, a , as:

$$u = (1 - a)V_0 \quad (2)$$

And by combining equations (1) and (2) we have:

$$u_1 = (1 - 2a)V_0 \quad (3)$$

And also the rotational velocity in the wake, C_θ , is given by defining tangential induction factor, a' :

$$C_\theta = 2a'wr \quad (4)$$

BEM method couple momentum theory with the events that are taking place around a real blade. Stream tube of 1-D momentum theory is divided into N annular element with thickness equal to dr , as shown in figure 2. Lateral boundary of this elements is limited by the stream lines. Therefore, no flow will pass through the elements.

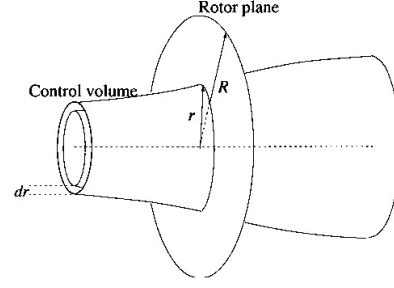


Fig. 2. Annular elements to be used in the BEM method [8]

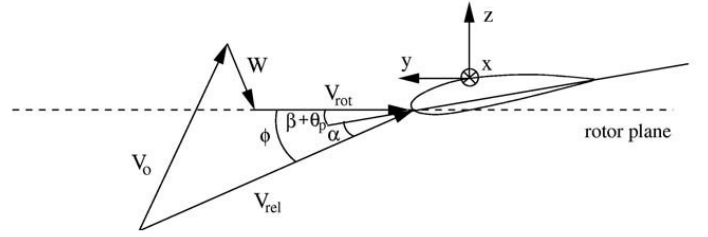


Fig. 3. Velocity triangle seen on a blade [8]

The following assumptions are used for annular elements in BEM method:

- There is no radial dependence, so what occurs in one element does not affect other elements.
- Force exerted by the blades on each of the annular elements is constant i.e. it is assumed that the number of rotor blades is infinite.

A correction factor known as Prandtl's tip loss factor is applied to reform the second assumption.

By using defined geometry, undisturbed wind velocity seen by the blade, V_0 , can be achieved by transforming wind velocity V_1 into the coordinate system 4:

$$\mathbf{V}_0 = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \mathbf{a}_{14} \mathbf{V}_1 \quad (5)$$

Where \mathbf{a}_{14} is the transformation matrix between coordinate system 1 and 4. As shown in figure 3 wind velocity seen by each blade V_{rel} is the summation of the rotational velocity, V_{rot} , the induced velocity, \mathbf{W} and V_0 :

$$\begin{pmatrix} V_{rel,y} \\ V_{rel,z} \end{pmatrix} = \begin{pmatrix} V_y \\ V_z \end{pmatrix} + \begin{pmatrix} -V_{rot} \\ 0 \end{pmatrix} + \begin{pmatrix} W_y \\ W_z \end{pmatrix} \quad (6)$$

From figure 3 it is obvious that the angle of attack, α , could be found when \mathbf{W} is known.

So by using BEM method the magnitude of induced velocities and angle of attack can be determined. For calculating induced velocities we have [9]:

$$W_n = W_z = \frac{-BL \cos \phi}{4\pi \rho r F |\mathbf{V}_0 + f_g \mathbf{n}(\mathbf{n} \cdot \mathbf{W})|} \quad (7)$$

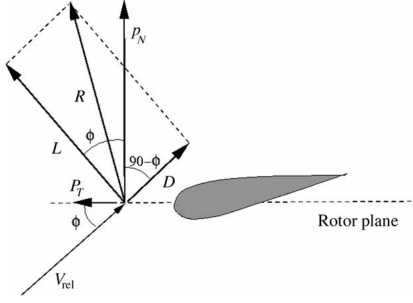


Fig. 4. The local loads on a blade [8]

and:

$$W_t = W_y = \frac{-BL \sin \phi}{4\pi \rho r F |\mathbf{V}_0 + f_g \mathbf{n}(\mathbf{n} \cdot \mathbf{W})|} \quad (8)$$

Where ρ is density of the air, r is radius of the blade, B is number of the blades, L is lift, F is Prandtl's tip loss factor and f_g is Glauert correction which would be explained later.

After calculating induced velocities magnitude the value of \mathbf{V}_{rel} can be determined. According to figure 3 we have:

$$\tan \phi = \frac{V_{rel,z}}{-V_{rel,y}} \quad (9)$$

So for angle of attack:

$$\alpha = \phi - (\beta + \theta_p) \quad (10)$$

Where β is twist angle and θ_p is pitch angle. Knowing α lift and drag coefficients could be determined by utilizing airfoil properties and:

$$L = \frac{1}{2} \rho |\mathbf{V}_{rel}|^2 c C_l \quad (11)$$

and:

$$D = \frac{1}{2} \rho |\mathbf{V}_{rel}|^2 c C_d \quad (12)$$

Where L and D are lift and drag and c is local chord. Since forces which are normal and tangential to the rotor plane are interested, as shown in figure 4 we project the lift and drag into these directions:

$$p_T = L \cos \phi + D \sin \phi \quad (13)$$

and:

$$p_N = L \sin \phi - D \cos \phi \quad (14)$$

Finally the generated power can be calculated by:

$$P = \omega p_N r \quad (15)$$

C. Correcting Models

1) *Prandtl's Tip Loss Factor*: As mentioned before Prandtl's tip loss factor, F , is a factor that corrects the assumption of infinite number of blades. It is defined as [8]:

$$F = \frac{2}{\pi} \cos^{-1}(e^{-f}) \quad (16)$$

where:

$$f = \frac{B}{2} \frac{R-r}{r \sin \phi} \quad (17)$$

B is number of blades, R is the maximum radius of each blade, r is the radius of each annular element and ϕ is the angle between relative velocity and horizontal line (as shown in figure 3)

2) *Glauert Correction*: When the axial induction factor become greater than a certain value, this method could not be used anymore. So a correction factor can be applied when induction factor is greater than a_c and defined as [8]:

$$f_g = \begin{cases} 1 & a \leq a_c \\ \frac{a_c}{a} (2 - \frac{a_c}{a}) & a > a_c \end{cases} \quad (18)$$

The value of a_c usually considered equal to 0.2.

3) *Dynamic Wake Model*: Due to effects of unsteadiness, during calculation of induced velocity some errors would appear. So it is necessary to use a filter for induced velocity to correct it's errors. One model which utilizes two first order differential equations is presented [10]:

$$W_{int} + \tau_1 \frac{dW_{int}}{dt} = W_{qs} + k \cdot \tau_1 \frac{dW_{qs}}{dt} \quad (19)$$

and:

$$W + \tau_2 \frac{dW}{dt} = W_{int} \quad (20)$$

Where W_{qs} is the value found by equations (7) and (8), W is the filtered value of induced velocity and τ_1 and τ_2 are two time constants calculating by:

$$\tau_1 = \frac{1.1}{(1 - 1.3a)} \cdot \frac{R}{V_0} \quad (21)$$

and:

$$\tau_2 = (0.39 - 0.26 \left(\frac{r}{R}\right)^2) \cdot \tau_1 \quad (22)$$

And k is a constant equal to 0.6.

4) *Yaw/Tilt Correction Model*: When the rotor is yawed (or tilted) the value of induced velocity differs from which calculated by equation (19) and (20). So it is necessary to use a model to correct this difference as [11]:

$$\mathbf{W} = \mathbf{W}_0 \left(1 + \frac{r}{R} \tan\left(\frac{\chi}{2}\right) \cos(\theta_{wing} - \theta_0)\right) \quad (23)$$

Where the wake skew angle, χ , is defined as the angle between the wind velocity in the wake and the rotational axis of the rotor by:

$$\cos \chi = \frac{\mathbf{n} \cdot \mathbf{V}'}{|\mathbf{n}| \cdot |\mathbf{V}'|} \quad (24)$$

And W_0 is the induced velocity found by equations (19) and (20) and θ_{wing} shows the positions of blade and is defined as:

$$\theta_{wing}(t + \Delta t) = \theta_{wing}(t) + \omega \cdot \Delta t \quad (25)$$

it's initial value is θ_0 .

D. Wind Velocity Profile

Wind velocity profile which is commonly used, is the profile obeying the boundary shear model as shown in figure 5. The equation for this profile is:

$$V_0(x) = V_0 \left(\frac{x}{H}\right)^\nu \quad (26)$$

where H is the hub height, x is the distance from the surface and ν a parameter giving the amount of shear and for this case is equal to 0.25.

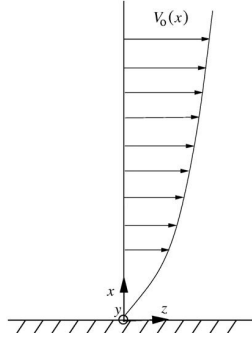


Fig. 5. Wind velocity profile [8]

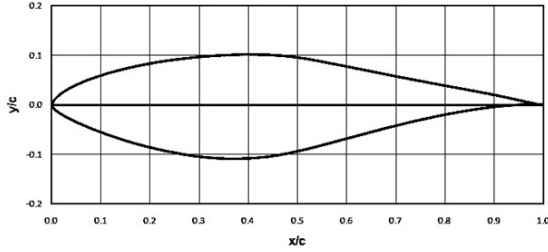


Fig. 6. NREL S809 airfoil configuration [12]

E. Calculation Algorithm

In order to calculate generated power, it is necessary to provide an algorithm and apply it at any moment for each blade element. Here we presented an algorithm to use. it consists of these steps:

- Defining geometry and parameters
- Initializing position and velocities
- Discretizing each blades into N elements
- Utilizing equation (6) to compute relative velocity
- Determining lift and drag coefficient using airfoil properties with known angle of attack
- Calculating normal and tangential forces
- Calculating generated power
- Utilizing equations (7) and (8) to find new values of unsteady induced velocities
- Utilizing correction models to find induced velocities' corrected values.

III. RESULTS

As mentioned before, the first step to simulate the performance of a wind turbine is to determining characteristics of turbine and it's surrounding air. So, for this we utilized characteristics which have been used in [8] for both turbine and it's blades and surrounding air. Also the airfoil section is a NREL S809 which is shown in figure 6. Properties of this airfoil are also shown in figure 7 [12].

First, we chose some initial and arbitrary values for different angles of turbine. These values are listed in table I. The turbine's performance is demonstrated and it's results are shown in figures 8, 9 and 10. In figure 8 exerted momentum

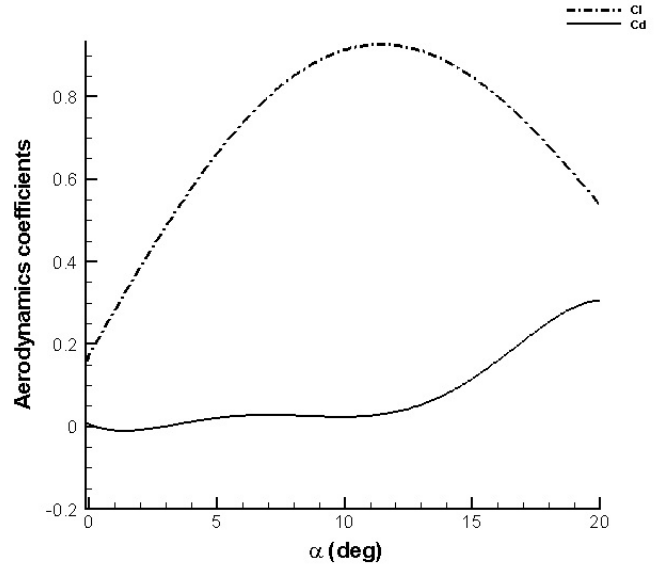


Fig. 7. Airfoil lift and drag coefficient against angle of attack

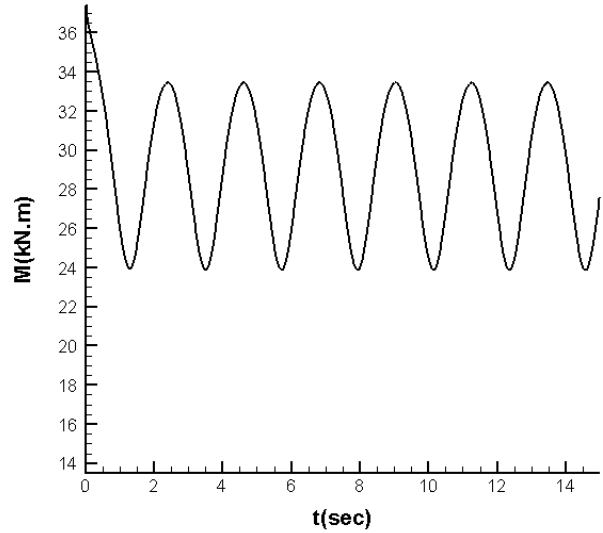


Fig. 8. Exerted momentum on each blade at different moments (wind velocity 12 m s^{-1})

on each blade plotted at different moment when velocity of upstream wind is equal to 12 m s^{-1} . Fluctuating behavior of this diagram is due to the different position of blade at each moment. Turbine generated power against time is plotted in figure 9 at upstream wind velocity equal to 12 m s^{-1} . As rotation of blades changes their whole position at any moment, generated power fluctuates in time. In figure 10 generated power calculated at different upstream wind velocity. As we expected in high wind velocity, increasing behavior of the generated power diagram ceases and turbine stalls.

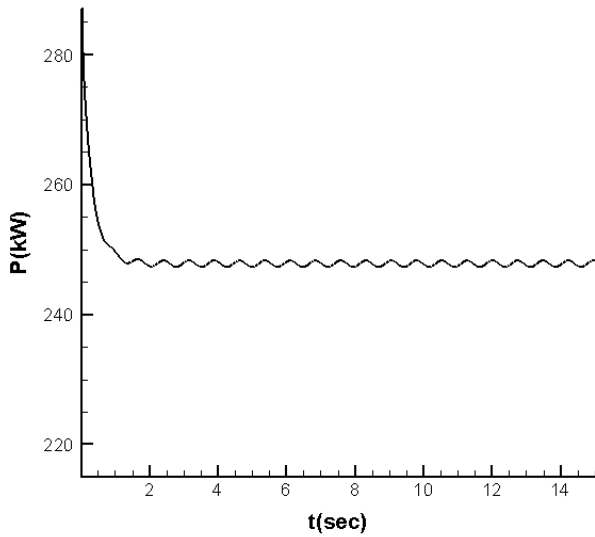


Fig. 9. Turbine generated power against time at wind velocity 12 m s^{-1}

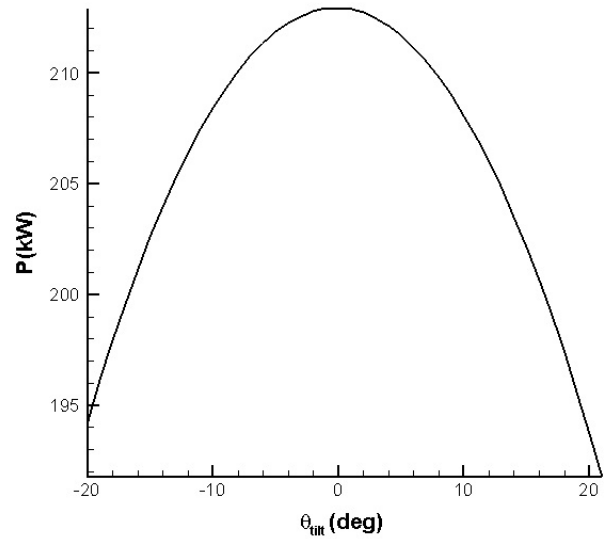


Fig. 11. Turbine generated power against tilt angle

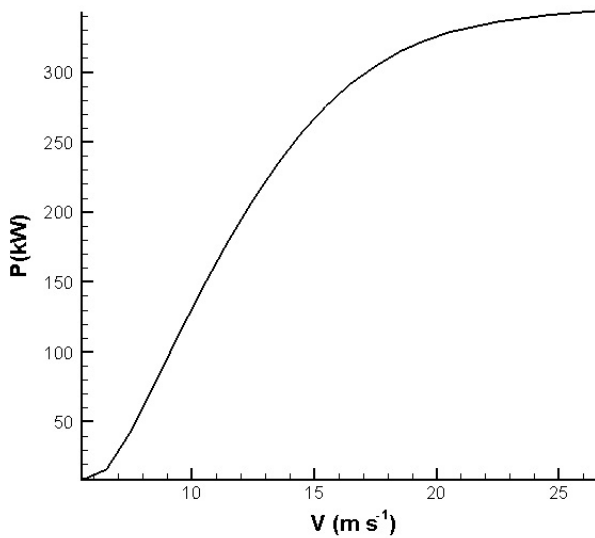


Fig. 10. Turbine generated power against wind velocity for arbitrary chosen turbine angles

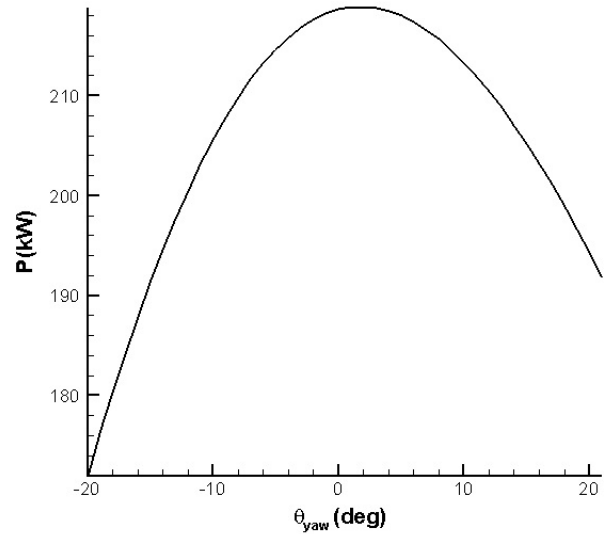


Fig. 12. Turbine generated power against yaw angle

Since we were interested in appropriate values of turbine angles, we changed each angle separately to calculate the generated power against different values of each angles in a certain velocity of upstream wind. Results are shown in figures 11, 12 and 13 for different values of θ_{tilt} , θ_{yaw} and θ_{cone} respectively.

Figure 11 shows the the generated power against different values of tilt angle. Since there is a point in this diagram that generated power is maximum on it, we chose this value for tilt angle as an appropriate value. Figure 12 has a maximum point

similar to figure 11. So this would be the appropriate value for yaw angle. By taking a look on figure 13 it is obvious that the generated power become maximum in two points. These two points were chosen as appropriate values for cone angle.

Appropriate values for each angle are listed in table I. The expectation was by choosing these values for turbine angles the generated power became maximum. Finally the magnitude of generated power calculated by choosing these new values for each angle is shown in figure 14 against wind velocity in upstream and comprised with it's initial magnitude.

The importance of choosing appropriate value for these

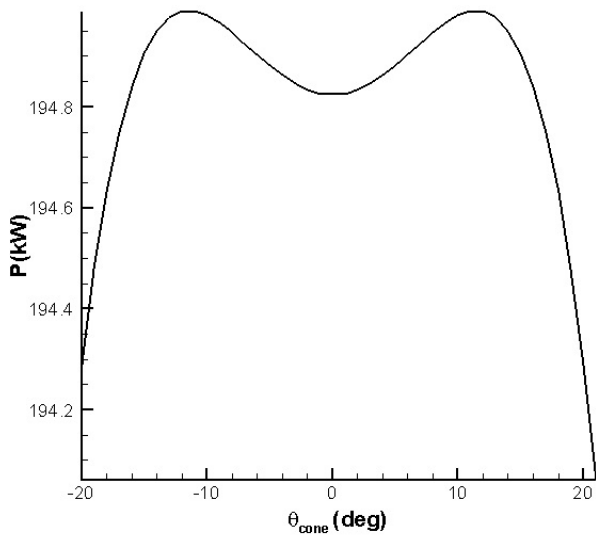


Fig. 13. Turbine generated power against cone angle

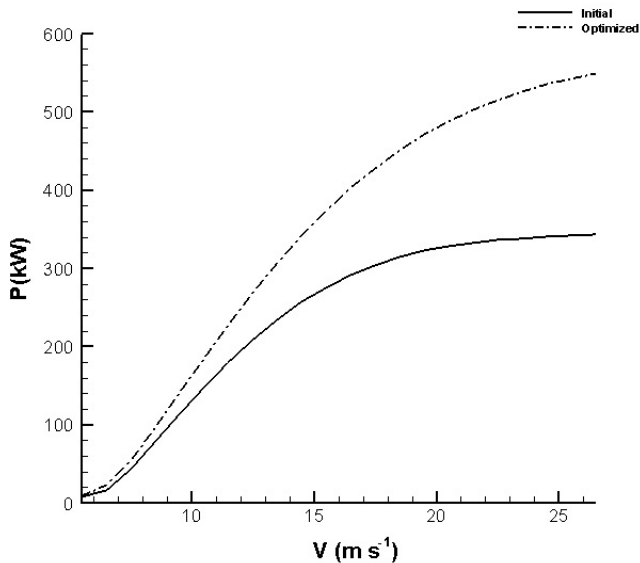


Fig. 14. Turbine generated power against wind velocity for both arbitrary chose turbine angles and calculated turbine angles

TABLE I
VALUES FOR WIND TURBINE ANGLES

Angle	Tilt	Cone	Yaw
Initial value (deg)	20	-20	-20
Appropriate value (deg)	0.0	0.0	± 11.45

angles can be inferred easily by investigating figure 14. It is obvious that generated power by turbine is increased approximately up to 2 times more, only by using appropriate values for these angles.

IV. CONCLUSION

Wind power is the conversion of wind energy into a useful form of energy and wind turbine is a device that converts wind's kinetic energy into electrical energy. Investigation of wind turbine performance by simulating it's behaviour is a necessary step before fabrication. An issue during design of wind turbine is the appropriate value for it's geometrical angles. In this study we demonstrated the appropriate values for these geometrical angles by simulating wind turbine behaviour. For this, unsteady blade element momentum (BEM) method was utilized. A certain value for each angle achieved. Finally the effect of choosing these new values on the turbine's performance calculated.

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